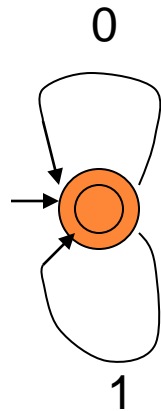
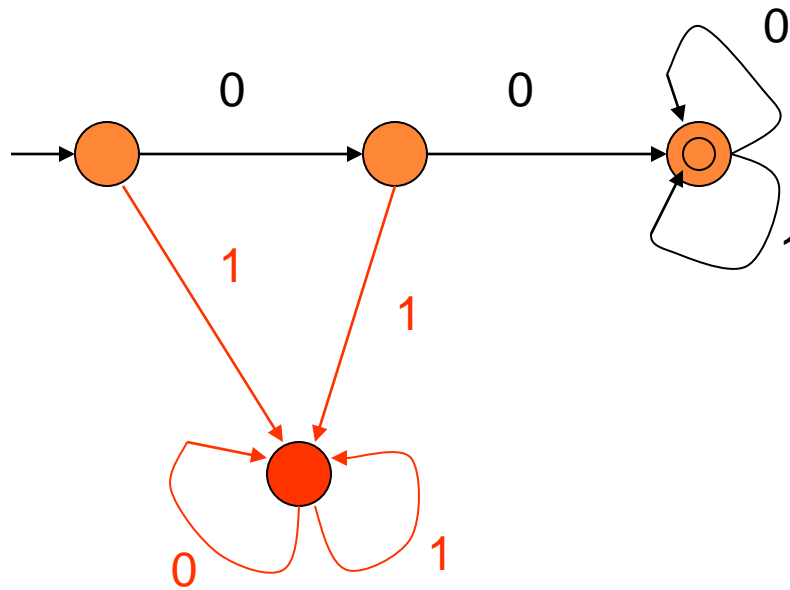


DFA

CONSTRUCT DFA TO ACCEPT $(0+1)^*$

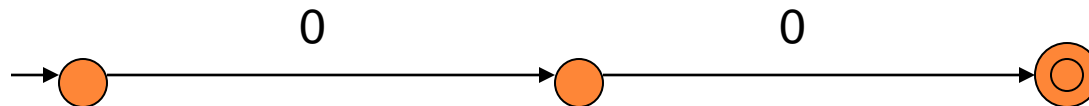


CONSTRUCT DFA TO ACCEPT $00(0+1)^*$



$(0+1)^*00(0+1)^*$

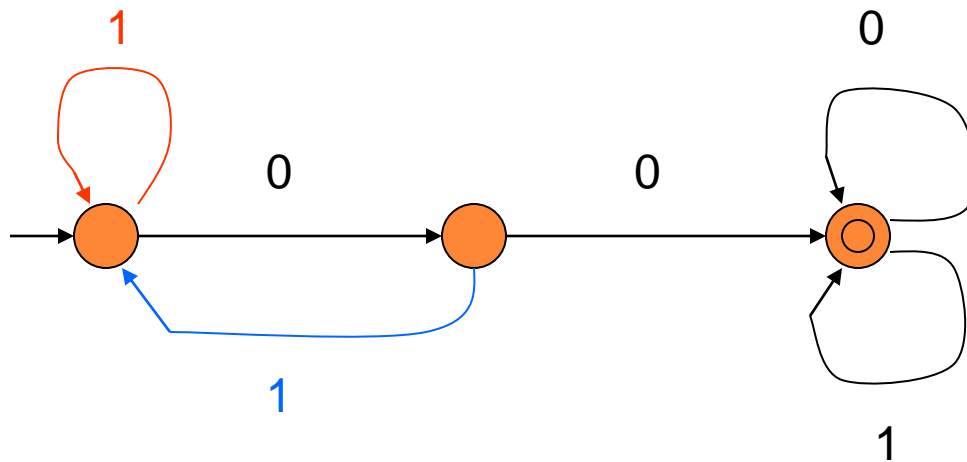
- **Idea:** Suppose the string $x_1x_2 \dots x_n$ is on the tape. Then we check x_1x_2 , x_2x_3 , ..., $x_{n-1}x_n$ in turn.
- **Step 1. Build a checker**



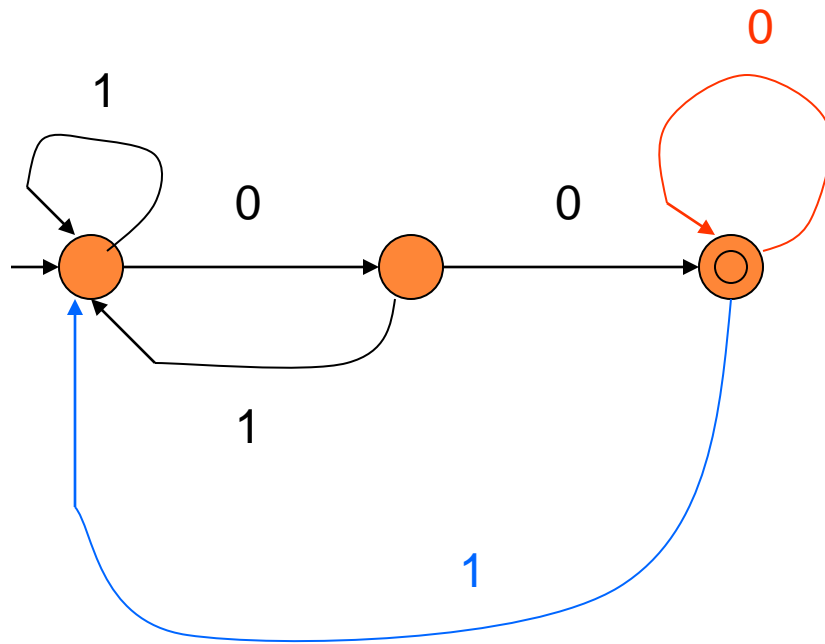
- Step 2. Find all edges by the following consideration:
Consider x_1x_2 .
- If $x_1=1$, then we give up x_1x_2 and continue to check x_2x_3 .
So, we have $\delta(q_0, 1) = q_0$.
- If $x_1x_2 = 01$, then we also give up x_1x_2 and continue to check x_2x_3 . So,
$$\delta(q_1, 1) = \delta(q_0, 1) = q_0.$$
- If $x_1x_2 = 00$, then $x_1x_2 \cdots x_n$ is accepted for any $x_3 \cdots x_n$. So,
 $\delta(q_2, 0) = \delta(q_2, 1) = q_2.$



$(0+1)^*00(0+1)^*$



$(0+1)^*00$

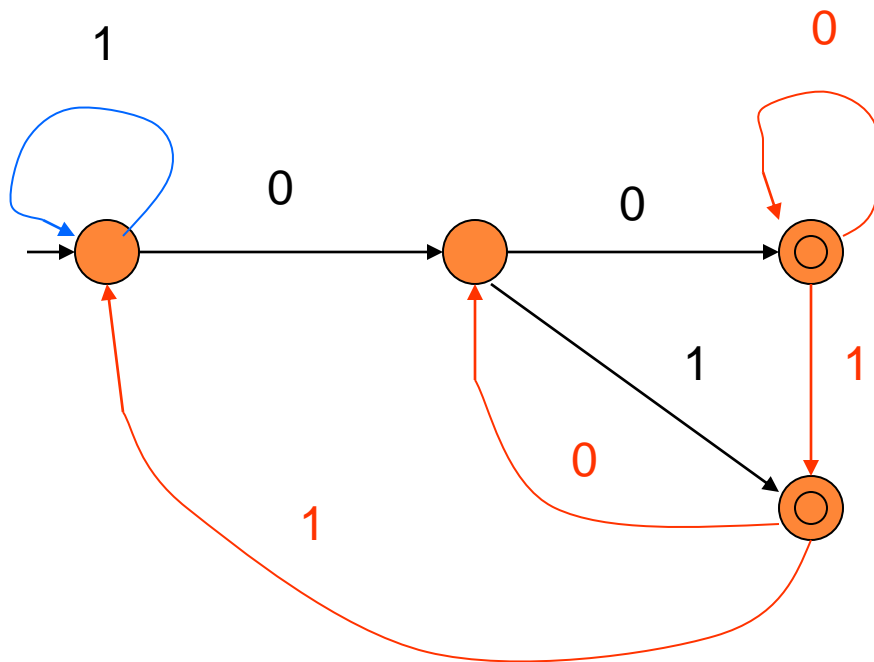


In general, suppose $x_1 x_2 \dots x_n$ is the string on the tape
And we want check $x_1 \dots x_k, x_2 \dots x_{k+1}, \dots$, in turn. If $x_1 \dots x_k$
Is not the substring that we want and we need to check
 $x_2 \dots x_{k+1}$, then we set that for and $i \leq k$,

$$\delta(\delta(s, x_1 \dots x_i), x_{i+1}) = \delta(s, x_1 \dots x_i x_{i+1}).$$



$(0+1)^*(00+01)$



2ND SOLUTION-PARALLEL MACHINE

$$L(M) = (0+1)^*00 \quad M=(Q, \Sigma, \delta, s, F)$$

$$L(M') = (0+1)^*01 \quad M'=(Q', \Sigma, \delta', s', F')$$

$$L(M'') = (0+1)(00+01) = L(M) \cup L(M')$$

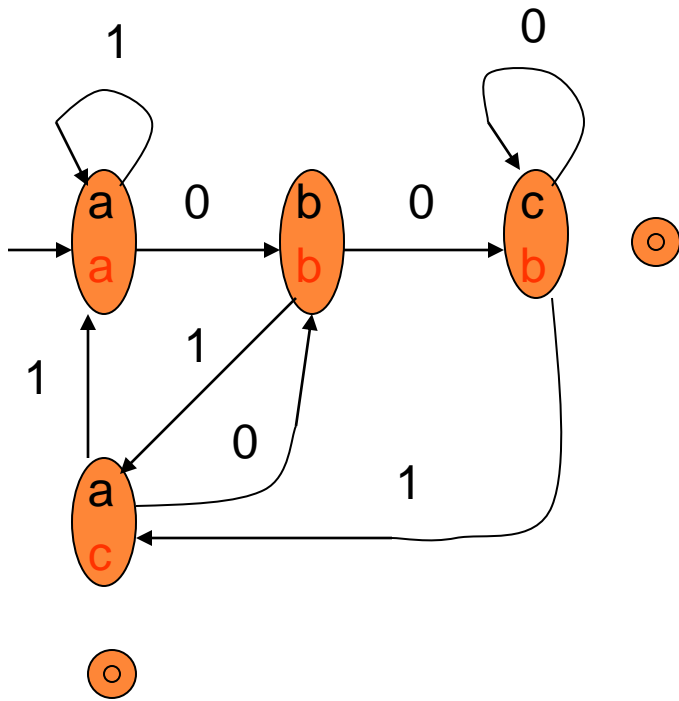
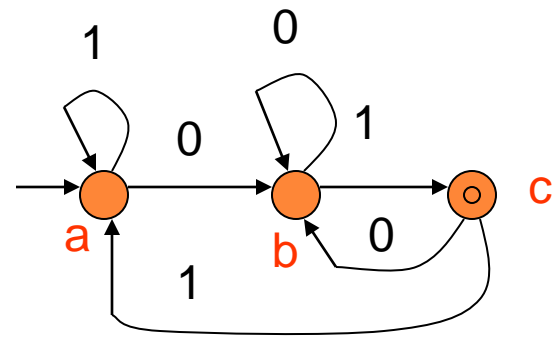
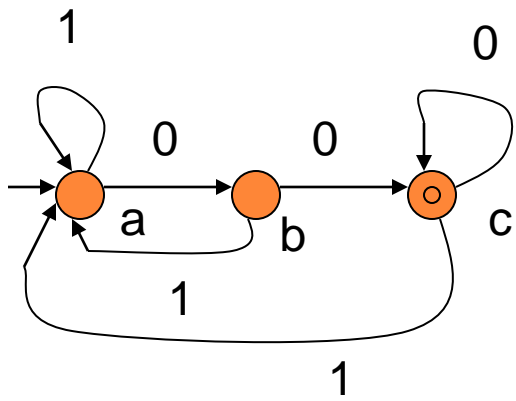
$$Q'' = \{(q, q') \mid q \text{ in } Q, q' \text{ in } Q'\}$$

$$\delta''((q, q'), a) = (\delta(q, a), \delta'(q', a))$$

$$s'' = (s, s')$$

$$F'' = \{(q, q') \mid q \text{ in } F \text{ or } q' \text{ in } F'\}.$$





UNION, INTERSECTION, SUBTRACTION

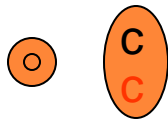
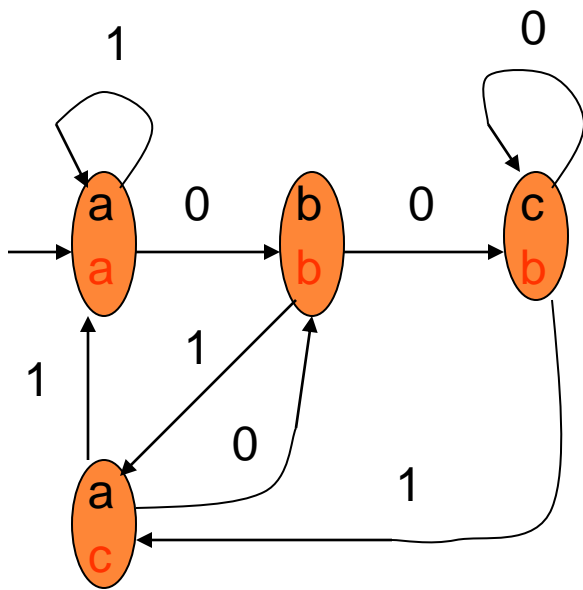
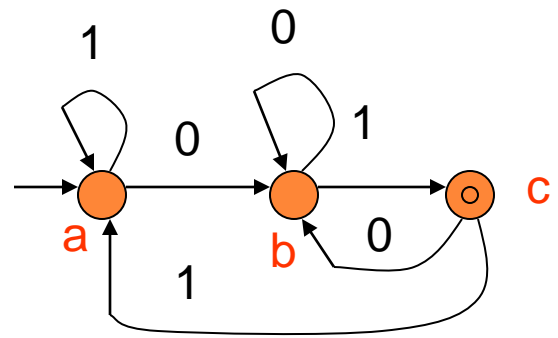
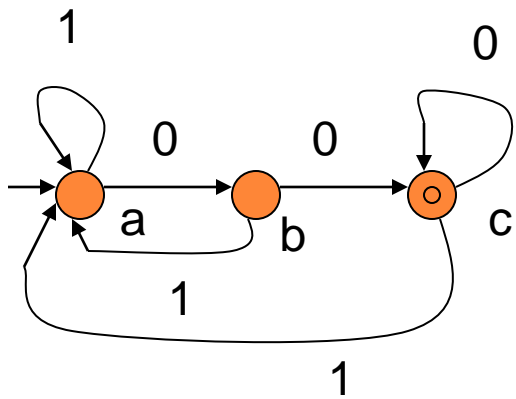
Union: $F'' = \{(q, q') \mid q \text{ in } F \text{ or } q' \text{ in } F'\}$

Intersection: $F'' = \{(q, q') \mid q \text{ in } F \text{ and } q' \text{ in } F'\}$

Subtraction: $L(M) \setminus L(M')$

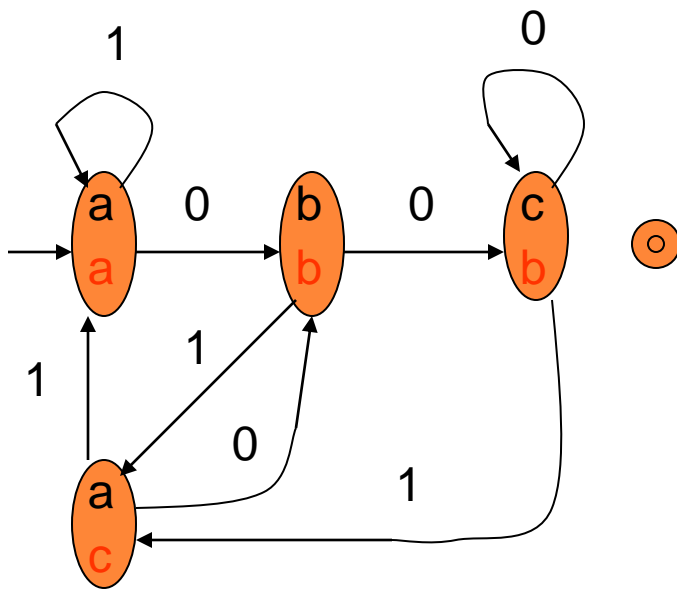
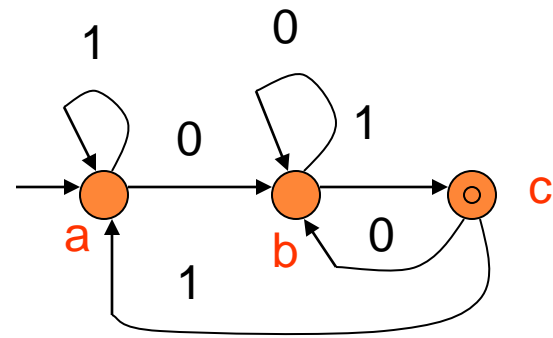
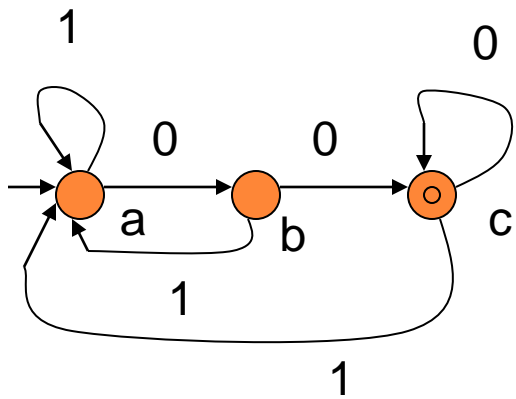
$F'' = \{(q, q') \mid q \text{ in } F \text{ and } q' \text{ not in } F'\}$





$$(0+1)^*00 \cap (0+1)^*01$$





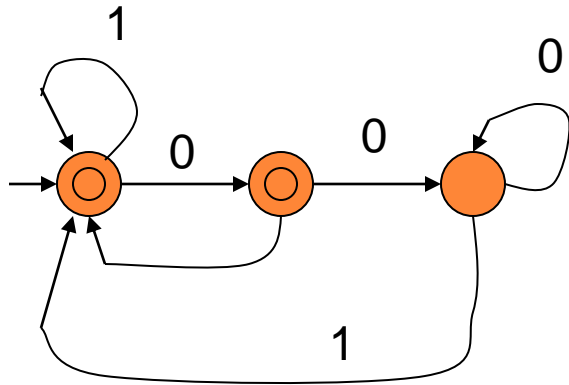
$$(0+1)^*00 \setminus (0+1)^*01$$



COMPLEMENT

○ $L(M') = \Sigma^* - L(M)$

$$F' = Q - F$$



$$\overline{(0+1)^*00}$$



THE SET OF ALL BINARY STRINGS BEGINNING WITH 1 WHICH INTERPRETED AS A BINARY REPRESENTATION OF AN INTEGER, IS CONGRUENT TO ZERO MODULO 5.

- **Idea:** To construct a DFA, we need to figure out **what should be states** and **where each edge should go**. Usually, **states can be found through studying the condition for accepting**. The condition for accepting a string $x_1x_2\cdots x_n$ is $x_1x_2\cdots x_n \equiv 0 \pmod{5}$. So, we first choose residues 0, 1, 2, 3, 4 as states.



$$2i + 0 \equiv ? \pmod{5}$$

$$2i + 1 \equiv ? \pmod{5}$$

